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Question $P_1 = x + y - z = 4$
$P_2 = 2x + y + z = 6$

do the two planes meet or intersect in a line?

Rules

1. $N_1 \times N_2$ = Dimentional vector of the line of intersection
2. $z = 0$ to find a point on the intersection line.

step 1: Find $N_1 \times N_2$

$N_1 = \langle 1, 1, -1 \rangle$
$N_2 = \langle 2, 1, 1 \rangle$
$N_1 \times N_2 = D = \langle 2, -3, -1 \rangle$

step 2: Find point

$z = 0 \quad \therefore \quad x + y = 4$
$2x + y = 6$

point $= (2, 2, 0)$

Step 3: Find line

point $= (2, 2, 0)$
$D = \langle 2, -3, -1 \rangle$

$$\therefore L : \left. \begin{aligned} x &= 2t + 2 \\ y &= 3t + 2 \\ z &= -t \end{aligned} \right\} t \in \mathbb{R}$$

Question: Find if 2 Planes are parallel.

Rules

1. $N_1 = c N_2$
2. $N_1 \times N_2$ = 0 vector (crossproduct = 0)
3. choose point Q_1 on P_1 and check if its on P_2

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$P_1 = \quad x + 2y + z = 1$

$P_2 = \quad 2x + 4y + 2z = 5$

step 1: $N_1 = C N_2$

$N_1 = \langle 1, 2, 1 \rangle$

$N_2 = \langle 2, 4, 2 \rangle$

$N_2 = 2N_1$

Note: if i cant determine if $N_2 = C N_1$ then Ido the cross product for a vector.

$N_1 \times N_2 = \langle 0\,0\,0 \rangle$

step 2: point on Q_1 on both P

1. substitute x and y as 0 in P_1

$\therefore z = 1$

point $= (0, 0, 1)$

2. substitute $x = 0$, $y = 0$, $z = 1$ in P_2 and see if left side $= 5$

(if it doesn't $= 5$ $\therefore$ they are parallel and Q_1 doesn't lie on P_2)

~~other way to check if p is on~~ P.

To check if 3 points are colinear

$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}$ must = 0 vector $\langle 0, 0, 0 \rangle$

Symmetric Equations: The equation of a line in terms of x, y and z.

eg if L: $\left.\begin{aligned} x &= 2t + 1 \\ y &= -3t + 4 \\ z &= 5t + 2 \end{aligned}\right\} t \in \mathbb{R}$

to find symmetric equation of L, solve for t

$\therefore \quad t \doteq \left(\dfrac{x-1}{2}, \dfrac{y-4}{-3}, \dfrac{z-2}{5} \right)$

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To check if a line lies on a plane.

subshitute the parametrix of the line into x, y and z of the plane.

(if left side = the constant on the right
∴ it lies on P)

to find intersection of line and a plane.

1. subshitute parametrix into P
2. solve for t
3. subshitute t into parametrix

∴ (x, y, z) is intersection point.

Derivatives.

$f'(x)$ is a function

$$= \frac{\lim f(x + \Delta x) - f(x)}{\Delta x}$$

$\Delta x \to 0$ ∈ (meaning t is very small)

Rules.

1. if $f(x) = c$ then $f'(x) = 0$
2. if $f(x) = cx^n$ then $f'(x) = cnx^{n-1}$
3. if $f(x) = \text{term}_1 \pm \text{term}_2 \pm \text{term}_3$ then $f'(x) =$ each term seperatly the $\pm$ depending on $f(x)$

example: $f(x) = 3\sqrt{x}$

$$= 3x^{1/2}$$

$$= \frac{3}{2} x^{-1/2}$$

Q. Find the tangent line of the point when $x = 2$ on $f(x)$

1st: find the derivative of the function.

2nd: substitute $x = 2$ into $f'(x)$ for the slope

3rd: use $y = mx + b$ where m is slope ↑

4rth: substitute the point $(2, y_0)$ (find y_0 by substituting $x = 2$ into $f(x)$) ∴ $y_0 = (\text{Slope})2 + b$

5th: Find b and write the formula of the line

Q $f(x) = x^3 - 6x^2 \overset{36}{\cancel{+16}}\, x + 1$

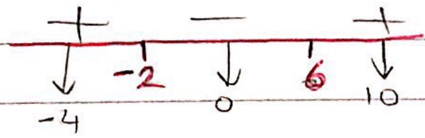
- find the sign of $f'(x)$
- identify when it's increasing / decreasing.
- sketch the graph.

1st: let $f'(x) = 0$

$$f'(x) = 3x^2 - 12x - 36 = 0$$
$$3(x^2 - 4x - 12) = 0$$
$$x^2 - 4x - 12 = 0$$
$$(x - 6)(x + 2) = 0$$

critical values. → ∴ $x = 6$ and -2

2nd: plot cv on an x-axis. and chose numbers inbetween to substitute them into $f'(x)$

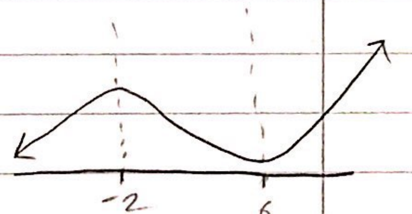


$f'(-4) = +$

$f'(0) = -$

$f'(10) = +$

graph:



∴ $f(x)$ is increasing from $(-\infty, -2) \cup (6, +\infty)$ and decreasing from $(-2, 6)$

Power formula / chain Rule

$u =$ a function $(x^2 + 3x)$ etc.

Rule $= nm(u)^{n-1} \cdot u'$

Q. $y = 10(2x^3 + x - 1)^{12} + 3x^4 - 2x^3 + 1$

$$y' = 120(u)^{11} \cdot u' + 12x^3 - 6x^2$$
$$= 120(2x^3+x-1)^{11} \cdot (6x^2+1) + 12x^3 - 6x^2$$

Q $y = \dfrac{3}{\sqrt[5]{(2x^3+2x-1)^2}} + 7(2x-1)^3 + 2$

$$y = \frac{3}{\sqrt[5]{u^2}} + 7u_2^3 + 2$$
$$= 3u^{-2/5} + 7u_2^3 + 2$$
$$y' = \frac{-6}{5} u^{-2/5 - 1} \cdot u' + 21u_2^2 \cdot u_2'$$
$$= -\frac{6}{5}(2x^3+2x-1)^{-2/5-1} \cdot (6x^2+2) + 21(2x-1)^2 \cdot (2)$$

Q. $f(x) = 2x^3 - 5x + 2$

a) find $(f(3x^2+1)) = k(x)$

b) find $(f'(3x^2+1))$

a) $2(3x^2+1)^3 - 5(3x^2+1) + 2$

b) $6(u)^2 \cdot u' - 5u^0 \cdot u$

$= 6(3x^2+1)^2 \cdot 6x - 5 \cdot 6x$

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If $k(x) = f(a(x))$
then $k'(x) = f'(a(x)) \cdot a'(x)$

given $k(x) = f(7x^2 - x + 1)$
~~and~~ $f'(7) = 10$ find $k'(1)$

1st: write the formula.

$k'(x) = f'(7x^2 - x + 1) \cdot (14x - 1)$

2nd: substitute 1 into x

$k'(1) = f'(7(1)^2 - 1 + 1) \cdot (14(1) - 1)$
$= f'(7) \cdot (13)$

$f'(7) = 10$

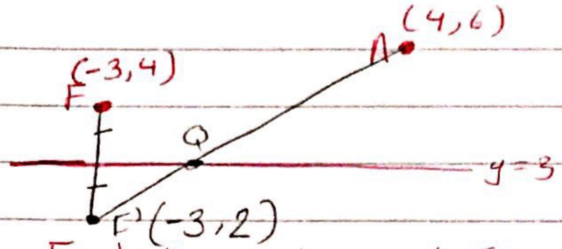
$\therefore \; k'(1) = 10 \cdot 13$
$= 130$ //

Q. $k(x) = 30 f(\sqrt{x+1} + 3)$
$f'(5) = -6$ find $k'(3)$.

$k'(x) = 30 f'((x+1)^{1/2} + 3) \cdot (\frac{1}{2}(x+1)^{-1/2})$

$k'(3) = 30 f'((3+1)^{1/2} + 3) \cdot (\frac{1}{2}(3+1)^{-1/2})$
$= 30 f'(5) \cdot (\frac{1}{4})$
$= 30 \cdot -6 \cdot \frac{1}{4}$
$= -\frac{180}{4} = -45.$ //

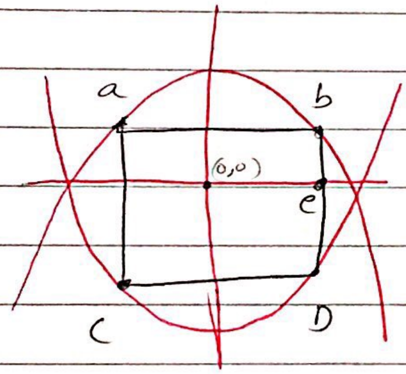
Scanned with CamScanner



Q Find the min point Q. on $y=3$

1st: project F or H and link ' to other.
3rd: The point where line meets $y=3$ is Q
4th: find equation of the line $y = mx + b$.

m from $\frac{y_2 - y_1}{x_2 - x_1}$ and
b by substituting
a point

Q


Q find the rectangle with biggest area.

$y = x^2 - 4$
$y = -x^2 + 4$

1st: write Area interms of 1 unknown.

$A = |ab| \, |bd|$
$= 2a(4 - e^2 - e^2 + 4)$
$= 2a(-2e^2 + 8)$
$= -4e^3 + 16e$

2nd: Derive A'

$A = -4e^3 + 16e$
$A' = -12e^2 + 16$

3rd: solve for e and substitute where you need it

$e = \sqrt{-16/-12}$

4th: to check use A'' (sub e into A'')
→ if + ∴ min
→ if − ∴ max

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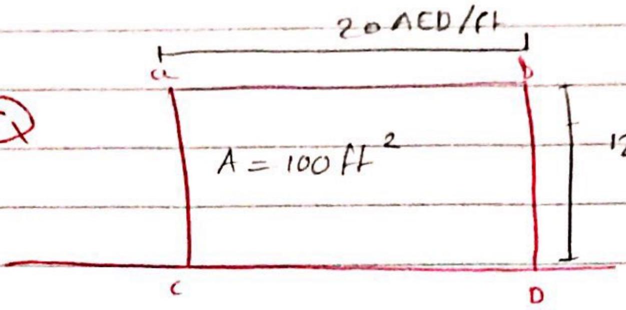
Q



Find minimum cost.

1st: write the formula of C in terms of 1 unknown.

$$C = 2(12w) + 20L$$

$$100 = Lw$$

$$L = \frac{100}{w}$$

$$C = 24w + 20\left(\frac{100}{w}\right)$$

2nd: Derive it

$$c' = 24 + -2000w^{-2}$$

$$24 = \frac{2000}{w^2}$$

$$w^2 = \frac{2000}{24}$$

$$w = 9.13$$

to check substitute 9.13 into c''

if $+$ $\therefore$ min //

product formula.

$$u'v + v'u$$

Q $y = \underbrace{(x^2 + 2x)}_{u}\underbrace{e^{(3x^2+2)}}_{v}$

$u' = 2x + 2 \qquad u = (x^2+2x)$

$v' = e^{(3x^2+2)} \cdot (6x) \quad v = e^{(3x^2+2)}$

$$y' = \left[(2x+2)\,e^{(3x^2+2)}\right] + \left[\left(e^{(3x^2+2)} \cdot (6x)\right)(x^2+2x)\right]$$

logarithmic equations.

properties:

1. $\ln 3 = \log_e 3 = m \quad \therefore e^m = 3$
2. $\log 3 = \log_{10} 3$
3. $\ln(ab) = \ln(a) + \ln(b)$
4. $\ln(a/b) = \ln(a) - \ln(b)$
5. $\ln a^n = n \ln a$
6. $\ln e = 1$

Deriving logarithems.

$y = Ce^{d(x)} \qquad y' = Ce^{d(x)} \cdot d'(x)$

$y = C\ln(d(x)) \qquad y' = \dfrac{C \cdot d'(x)}{d(x)}$

eg. $15 = e^{(2x+1)}$

$\ln 15 = \ln e^{2x+1}$

$\ln(15) = (2x+1) \cdot 1$

$x = \dfrac{\ln(15) - 1}{2}$

$= 0.85.$

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Math week 12 Sunday.

Q $y=(3x+1)e^{(2x+5)}$

$y' = u'v + v'u$

$= 3e^{(2x+5)} + \left[e^{(2x+5)} \cdot (2)(3x+1)\right]$

$y = \sqrt[3]{(x^2+1)^2} \cdot e^{(3x^2+6x-1)}$

$= (x^2+1)^{2/3} \cdot e^{(3x^2+6x-1)}$

$y' = u'v + v'u$

$= \left[\left(\frac{2}{3}(x^2+1)^{2/3-1} \cdot (2x)\right) \cdot e^{(3x^2+6x-1)}\right] + \left[e^{(3x^2+6x-1)} (6x+6) \cdot (x^2+1)^{2/3}\right]$

$y = \frac{x+1}{\sqrt{x^2+1}}$

$= (x+1) \cdot (x^2+1)^{-1/2}$

$y' = \left[1 \cdot (x^2+1)^{-1/2}\right] + \left[\left(-\frac{1}{2}(x^2+1)^{-3/2} \cdot 2x\right)(x+1)\right]$

$y = \frac{x^2+1}{e^{-3x}}$

$= (x^2+1) \cdot (e^{3x})$

$y' = \left[2x\,e^{3x}\right] + \left[3e^{3x} \cdot (x^2+1)\right]$

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$$y = C\ln(d(x))$$

$$y' = \frac{C \cdot d'(x)}{d(x)}$$

$$y = 10\ln(2x^3 + 2x - 1)$$

$$y' = \frac{10 \cdot (6x^2 + 2)}{(2x^3 + 2x - 1)}$$

$$y = \ln\left[\frac{3x+2}{15x^2+x-1}\right]$$

$$= \ln(3x+2) - \ln(15x^2 + x - 1)$$

$$y' = \left[\frac{3}{3x+2}\right] - \left[\frac{30x+1}{15x^2+x-1}\right]$$

$$y = \ln\left[\frac{(3x+1)^{2019}}{\cancel{7x^3-x+2}} \cdot\right]\sqrt{3x+2}\,\Big)$$

$$y = \ln(3x+1)^{2019} + \ln(3x+2)^{1/2}$$

$$= \left[2019\ln(3x+1)\right] + \left[\tfrac{1}{2}\ln(3x+2)\right]$$

$$= \left(\frac{2019 \cdot 3}{3x+1}\right) + \left[\frac{1}{2} \cdot \frac{3}{3x+2}\right]$$

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$y = 3x + 2e^{(x+1)} + 7$

① find tangent line of $f(x)$ (or y) when $x = -1$

② find the equation of the normal to the curve. when $x = -1$

T: $y = mx + b$

$m = f'(-1)$

$f'(x) = 3 + 2e^{(x+1)}$

$f'(-1) = 3 + 2e^{-1+1}$

$= 3 + 2$

$m = 5$

find point from $f(x)$ when $x = -1$

$y = 3(-1) + 2e^{(-1+1)} + 7$

$= -3 + 2 + 7$

$= 6$

point $= (-1, 6)$

T: $6 = 5(-1) + b$

$b = 11$

$y = 5x + 11$

N: $m = -\frac{1}{5}$

$6 = -\frac{1}{5}(-1) + b$

$b = 6 - \frac{1}{5}$

$= \frac{29}{5}$

N: $y = -\frac{1}{5} + \frac{29}{5}$

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